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**SECOND EDITION**

**Kevin B. Korb  
Ann E. Nicholson**



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*To Judea Pearl and Chris Wallace*

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## **Preface**

Bayesian Artificial Intelligence, in our understanding, is the incorporation of Bayesian inferential methods in the development of a software architecture for an Artificial Intelligence (AI). We believe that important ingredients of such an architecture will be Bayesian networks and the Bayesian learning of Bayesian networks (Bayesian causal discovery) from observation and experiment. In this book we present the elements of Bayesian network technology, automated causal discovery, learning probabilities from data, and examples and ideas about how to employ these technologies in developing probabilistic expert systems, which we call Knowledge Engineering with Bayesian Networks.

This is a very practical project, because data mining with Bayesian networks (applied causal discovery) and the deployment of Bayesian networks in industry and government are two of the most promising areas in applied AI today. But it is also a very theoretical project, because the achievement of a Bayesian AI would be a major theoretical achievement.

With our title there are a number of subjects we could naturally include, but have not. Thus, another necessary aspect of an effective Bayesian AI will be the learning of concepts, and hierarchies of concepts. Bayesian methods for concept formation exist (e.g., Chris Wallace's *Snob*, Wallace and Boulton, 1968), but we do not treat them here. We could also have discussed function discovery, polynomial curve fitting, time series modeling, etc. We have chosen to hew close to the theme of using and discovering Bayesian networks both because this is our own main research area and because, important as the other Bayesian learning methods are, we believe the Bayesian network technology is central to the overall project. We have added for this edition a treatment of the use of Bayesian networks for prediction (classification).

Our text differs from others available on Bayesian networks in a number of ways. We aim at a practical and accessible introduction to the main concepts in the technology, while paying attention to foundational issues. Most texts in this area require somewhat more mathematical sophistication than ours; we presuppose only a basic understanding of algebra and calculus. Also, we give roughly equal weight to the causal discovery of networks and to the Bayesian inference procedures using a network once found. Most texts either ignore causal discovery or treat it lightly. Richard Neapolitan's book, *Learning Bayesian Networks* (2003), is an exception, but it is more technically demanding than ours. Another distinguishing feature of our text is that we advocate a causal interpretation of Bayesian networks, and we discuss the use of Bayesian networks for causal modeling. We also illustrate various applications of the technology at length, drawing upon our own applied research. We hope that these illustrations will be of some interest and indicate some of the possibilities for the

technology. Our text is aimed at advanced undergraduates in computer science who have some background in artificial intelligence and at those who wish to engage in applied or pure research in Bayesian network technology.

A few remarks about notation before we begin. The notation special to Bayesian networks (or to our treatment of them) will be introduced as we proceed; first introductions of notation (including general mathematical notation) and acronyms will be recorded, with page numbers, in Appendix A. When first introducing new terminology we shall employ boldface to point it out; thus, for example, the first appearance (after this) of “Bayesianism” will be in boldface.

Here we describe the simplest aspects of the notation we adopt. First, variables (nodes in the network) will be named, with the names being capitalized and usually italicized (e.g., *Y*, *Alarm*, *Cancer*). Sets of variables will be set in boldface (e.g., **X, Y**). The values that variables take will not be capitalized, but will be italicized; thus, to assert that the alarm is on, we might write *Alarm = on*. Values abbreviated to single letters, such as *True (T)* and *False (F)*, however, will be capitalized. Where no confusion is likely to arise, variables and values may be abbreviated.

The book Web site is <http://www.csse.monash.edu.au/bai> and contains a variety of aids for study, including example Bayesian networks and data sets. Instructors can email us for sample solutions to many of the problems in the text.

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## New in the second edition

We have found and corrected numerous mistakes in the first edition, most of the credit for which belongs to readers and students (see below). We claim full credit for all the errors remaining or newly introduced. Other new introductions include: §4.7, object-oriented Bayesian networks, Chapter 7, Bayesian Network Classifiers, which describes naive Bayes models and other classifiers; §9.7, which addresses two foundational problems with causal discovery as well as Markov blanket discovery; §9.8, which treats methods of evaluating causal discovery programs; substantial new material in Chapter 10, including discussions of many common modeling errors; and new applications and case studies in Chapters 5 and 11. The uses of causal interventions to understand and reason with causal Bayesian networks receive fuller coverage in a number of places. The Evaluation Chapter of the first edition has been divided, enhanced and scattered: evaluation of prediction goes into Chapter 7, evaluation of causal discovery into Chapter 9 and sensitivity analysis and related issues into Chapter 10. The result is a somewhat thicker book and one we hope is more useful for students and practitioners.

---

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